Session 2
Weight and Balance

1.0 Definitions

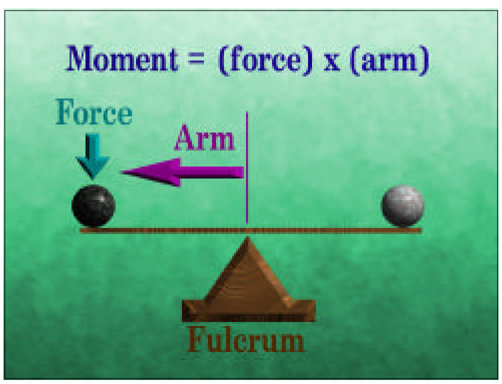
Center of gravity (*cg*) - The point about which the plane would balance if it were possible to suspend the plane at that point; the mass center of the plane at which the entire weight of the plane is assumed to be concentrated.

Center of gravity limits - The specified forward and aft points within which the *cg* must be located during flight.

Reference datum line (RDL) - An imaginary vertical line from which all arm measurements are taken.

Arm - The horizontal distance from the Reference Datum Line to the *cg of any particular item.*

Moment - The product of a force (or weight of an item) multiplied by its arm. The total moment of an object is the weight of the object multiplied by the length of the arm from the RDL to the *cg*.



Moments and Arms

Control - The ability to generate desired movements through the use of forces.

Fulcrum - The pivot point of a lever; balance point of a beam.

Longitudinal axis - An axis of rotation through the *cg* which runs from nose to tail of the aircraft. (Figure 2.1)

Lateral axis - An axis of rotation through the *cg* which runs from wingtip to wingtip of an aircraft. (Figure 2.1)

Directional axis - An axis of rotation perpendicular to the longitudinal and lateral axis which runs vertically through the center of gravity. (Figure 2.1)

NOTE:

All axes pass through the center of gravity and are perpendicular to each other at that point.

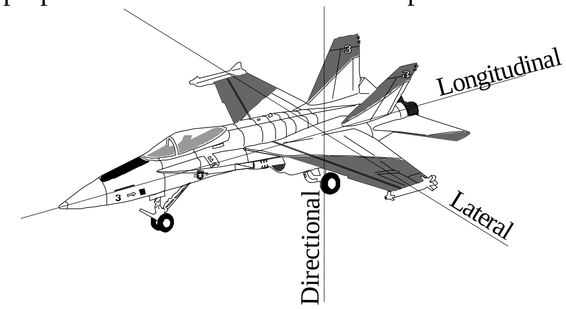


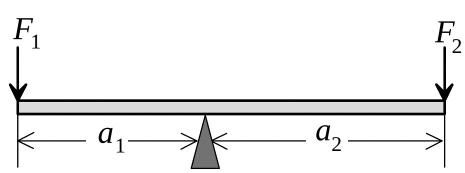
Figure 2.1 Aircraft Axis

NOTE:

Session 3 of the text should be reviewed prior to starting the video.

2.0 Balancing Forces and Moments

During the first session we determined that an object's weight is a measure of the force it exerts on the Earth. We also saw how according to Newton's third law, forces exist in "equal and opposite" pairs. Any time an out of balance force exists, there is an acceleration in the direction of the greater force. Many times when we use a see-saw we are faced with two forces (or weights) which are not equal. Then to level the board over the fulcrum, we used the concept of balanced moments, as shown in Figure 2.2



$F_1 a_1 = F_2 a_2$

Figure 2.2 Balanced Moments of a See-Saw

Recall that a moment is the product of a force multiplied by a distance, or arm. Therefore it stands to reason that a smaller force acting at a greater distance could quite easily balance a larger force acting at a smaller distance.

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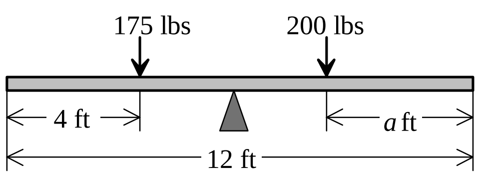
NOTE:

Mechanics often refer to a moment as a "torque" and will list automobile performance in the form of "foot-pounds of torque." Additionally, the "torque wrench" used in auto repair is simply the mechanic's arm strength applied over the known length of the wrench. A gauge indicates "foot pounds or inch - pounds" of torque based on the amount of force applied by the mechanic.

The following example highlights how moments are balanced.

***Example 1*:** A person weighing 175 pounds sits 4 feet from the end of a bench seat. A second person weighing 200 pounds wants to sit at the opposite end of the bench. If the fulcrum is in the middle of the 12 foot bench, how far from the end should the 200 pound person sit for the bench to remain level?



***Solution*:**

1. Calculate the moments on the left side of the fulcrum:

   force × *arm* = *moment*

   (175 lbs) × (6 ft – 4 ft) = 350 ft-lbs

Caution:

The question is asking how far from the END of the board should the person sit. Since in this case we are balancing the bench with reference to the fulcrum, and the fulcrum is in the middle of the board, the arm is subtracted from half the board length.

2. To balance the bench, the moments on the left must equal the moments on the right, therefore:

   $350\ ft\text{-}lbs = (200\ lbs) \times (6\ ft - a\ ft)$

   $350\ ft\text{-}lbs = 1200\ ft\text{-}lbs - 200\ lbs \times (a)ft$

   $(350\ ft\text{-}lbs) - (1200\ ft\text{-}lbs) = -200\ lbs \times (a)ft$

   $$\frac{-850ft - lbs}{-200lbs} = 4.25ft = (a)ft$$

3. Therefore, the 200 pound person should sit 4.25 feet from the right end of the bench.

4. If the fulcrum is in the middle, 6 ft are on each side. Therefore the distance from the fulcrum is 6 - 4.25 ft. = 1.75 ft.

Proof:

175 lbs. × 2 ft. = 200 lbs. × 1.75 ft

300 ft-lbs = 300 ft-lbs

In the above example, the arms were measured with respect to the pivot point. However when dealing with aircraft we're trying to FIND the balance point, or more specifically, the center of gravity. Therefore the arms are measured with respect to the reference datum line (RDL). This imaginary line is usually located at the nose of the aircraft and is used solely as a reference point for calculating the center of gravity.

3.0 Significance of Weight & Balance

Determining an aircraft's total weight and the location of the center of gravity is crucial to predicting the aircraft's performance and controllability. As we will see in future sessions, an increase in an aircraft's weight has a direct impact on the following areas of that plane's performance:

higher takeoff speed
longer takeoff run
longer landing roll

How the weight is distributed aboard an aircraft is in part determined by the pilot and in part determined by the designer. The pilot can affect how much fuel, people, and cargo is put onboard but the designer decides where the fuel, people, and cargo are placed. The designer's decisions are based upon being able to balance the airplane and control the aircraft's movements. Uppermost in the designer's concern for balancing the aircraft is the fore and aft location of the center of gravity along the longitudinal axis of the aircraft. Balancing the aircraft results when the sum of the moments around the center of gravity equals zero, often written as $\sum M_{cg} = 0$. However, balancing the aircraft on the lateral axis left and right of the longitudinal axis is also very important. Each item in an aircraft has weight and subsequently exerts a force at a specific location on the plane. Resolving these forces into one resultant force acting at a specific location will yield the center of gravity of the airplane.

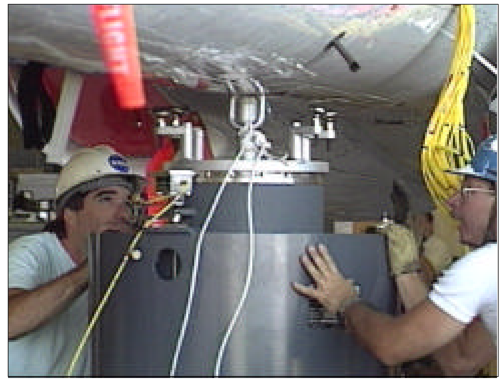
Caution:

The student should be familiar with the method involved in resolving parallel forces into a single resultant force. See Example 2.1 and 2.2 of the Operational Supplement for an explanation of resolution of forces.

The *cg* is not necessarily a fixed point for every loading condition; its location depends on the weight distribution of the aircraft. As fuel is burned throughout a flight or passengers change seats, the *cg* shifts accordingly. The designer has accounted for this movement to a certain degree by providing the pilot with a range of acceptable *cg* locations where aircraft control may be retained. The amount of control a pilot has is a function of the size of the control surfaces and how large of a moment these surfaces can generate. Control surfaces include the elevator on the tail which creates a force to rotate the aircraft about the lateral axis; ailerons on the wing which rotate the aircraft about the longitudinal axis; and the rudder on the tail which rotates the aircraft about the directional axis. In order to generate a moment, the control surface must create a force located at a distance from the center of gravity. When an aircraft is in flight, any force exerted by a control surface tends to rotate the aircraft around the center of gravity making knowledge of the *cg* location critical. Determining the location of the *cg* begins with weighing the aircraft.

4.0 Weighing An Aircraft

Determining the weight of an aircraft is simply a matter of summing forces. A scale is placed under each point where the aircraft touches the ground, and the readings of all the scales are then added together.



Weighing the Aircraft

Caution:

This may seem very basic however it is very important to place a scale under each point of ground contact. Failure to do so will result in an erroneous total.

Some aircraft have very unusual landing gear arrangements. For example, the U.S. Air Force B-52 bomber has an “outrigger” landing gear under each wingtip. These support the weight of the wings when they are full of fuel. Therefore, in order to get the total weight of a B-52, a scale would also have to be placed under each “outrigger” gear. The video depicts the procedures involved with weighing NASA’s F-18 High Angle of Attack Research Vehicle (HARV). The aircraft was lifted by a crane and large scales were placed under the landing gear.

Lifting Aircraft

Once the total weight of the aircraft is determined, determining the center of gravity location is accomplished through resolution of the forces into a resultant force acting at the *cg* of the aircraft.

****STOP VIDEO after two lines marked on metal****

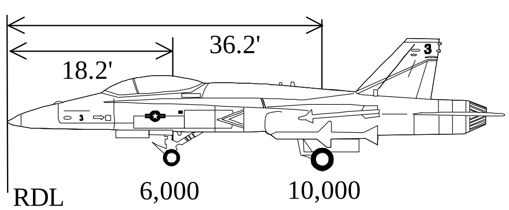
cg of irregular shape

NOTE:

The video also demonstrated an experimental way of determining the *cg* of an irregular shaped object. Further explanation may be found in the Operational Supplement for this session.

***Example 2*:** Determine the center of gravity location of the F-18 HARV given the following weights and arms:

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Nose Wheel	6,000	18.2	
Main Wheels (ea.)	10,000	36.2	
Total	**26,000**		



F-18 HARV Weights

Solution:

Using the relationship that the $cg = \frac{S_{moment}}{S_{weight}}$ we:

First, find the total weight and moment for the entire aircraft.

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Nose Wheel	6,000	18.2	109,200
Main Wheels (ea.)	10,000	36.2	362,000
Total	**26,000**		**833,200**

Second, by dividing the total moment by the total weight, the location of the *cg* is found.

$$\frac{833,200 ft - lbs}{26,000 lbs}$$

= 32.04*ft* from the Reference Datum Line

NOTE:

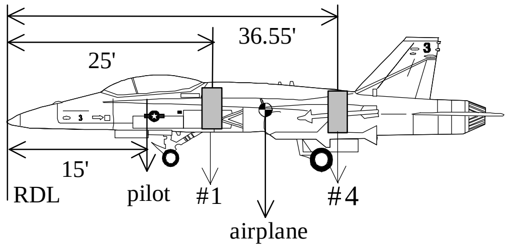
For the purposes of this example, the Reference Datum Line is assumed to be at the nose of the aircraft. The arm to the nose wheel is 18.2 feet and the arm to the main wheel is 36.2 feet. Therefore, the *cg* is 32.04 feet from the nose.

****START VIDEO****

As stated previously, the designer accounts for movement of the *cg* in flight by providing an acceptable *cg* range where control of the aircraft can be maintained. When fuel is burned, weight is removed, so there is less force acting at a given point on the aircraft. The *cg* location will therefore change. For the aircraft to be balanced in flight, the moments forward of the *cg* must be equally opposed by the moments aft of the *cg*. The following example will highlight how center of gravity moves in flight.

***Example 3*:** Given the following items and associated arm lengths, calculate how much the center of gravity moves when all of the fuel is burned from the # 1 fuel tank.

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Empty airplane	26,000	32.04	833,200
pilot	155	15.00	
#1 fuel tank	2,150	25.00	
#4 fuel tank	3,620	36.55	



F-18 HARV Loading Diagram

***Solution*:**
Again using the relation that the

$$arm_{cg} = \frac{total\ moment}{total\ weight}$$

1. Calculate the moment for each item:

pilot	155 lbs	× 15.0 ft	=	2,325	ft-lbs
#1	2,150 lbs	× 25.0 ft	=	53,750	ft-lbs
#4	3,620 lbs	× 36.5 ft	=	132,311	ft-lbs

2. Determine the current *cg* location

item	weight (lbs)	moment (ft-lbs)
Empty airplane	26,000	833,200
pilot	155	2,325
#1 fuel tank	2,150	53,750
#4 fuel tank	3,620	132,311
Total	**31,925**	**1,021,586**

$$\frac{1,021,586 ft-lbs}{31,925 lbs}$$

= 32 feet from the Reference Datum Line

NOTE:

As you can see even adding fuel and pilot the *cg* only moved 0.4 feet (4.8 inches) compared to the over all length of the airplane. This is negligible.

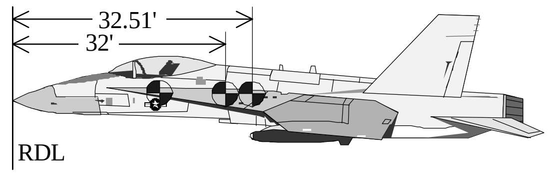
3. Determine how much the *cg* moves when the fuel in # 1 tank is burned off. To do this, simply subtract the weight of the fuel in # 1 tank from the total weight, and subtract the moment from the total moment. Then calculate the new *cg* by dividing the new moment by the new weight.

$$31,925\ lbs - 2150\ lbs = 29,775\ lbs$$

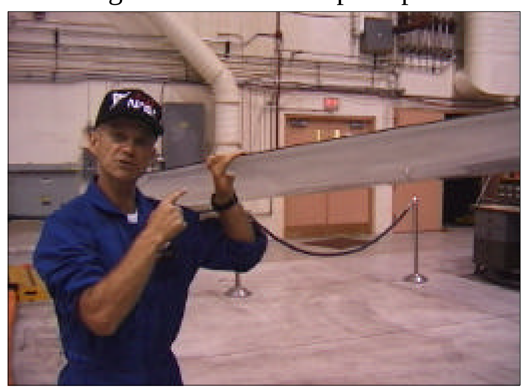
$$1,021,586\ ft\text{-}lbs - 53,750\ ft\text{-}lbs = 967,836\ ft\text{-}lbs$$

New *cg* location is

$$\frac{967,836 ft-lbs}{29,775 ft} = 32.51 ft$$



The movement of the center of gravity may not appear to be very significant, however if the airplane is to stay balanced (remain in level flight) the force that has been lost due to removal of the fuel weight, must be replaced with a force created by the tail. If the amount of weight removed cannot be replaced by a force generated by the tail, the aircraft experiences what is termed “loss of control authority.” What this really means is the *cg* has moved to a location where the force created by the tail is no longer sufficient to keep the plane level.



Downforce Created by Tail

Consider the following:

***Example 4*:** Assuming the fuel has burned out of Tank # 1, as shown in Example 3, how much force must be generated by the tail to keep the *cg* in the same location (32 feet) if the tail is located 51 feet from the Reference Datum Line?

***Solution*:**

Using the basic relationship $arm_{cg} = \frac{total\ moment}{total\ weight}$

1. Determine the total moment lost when the fuel in # 1 tank burned.

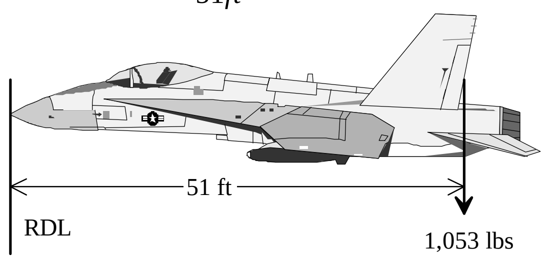
item	weight (lbs)	arm (ft)	moment (ft-lbs)
#1 fuel tank	2,150	25.00	53,750

2. Determine the tail force required.

The moment which must be replaced to keep the *cg* at 32 feet is found by step 1. Since the tail has an arm of 51 feet, the force is found by dividing the moment needed by the arm length.

$$\frac{moment}{arm} = force$$

$$\frac{53,750ft - lbs}{51ft} = 1053lbs$$



Then to keep the aircraft in level flight, the tail must generate 1053 pounds of force. The total amount of force a tail can generate is based on a number of factors including the speed, distance of the tail from the reference line, and the size of the tail. Based on these factors the designer sets the amount the *cg* can move in flight since he has calculated the maximum amount of up (or down) force the tail can generate. A force generated by an aerodynamic surface, such as a wing or tail, is termed lift. How lift is generated is the subject of the next session.

5.0 Measures of Performance

1 What is a moment?

2 What is the relationship between the flight control surfaces and the *cg*?

6.0 Suggested Activity

1 A suggested activity is to have each student determine the center of gravity of an object found in the classroom by both the experimental and the analytical methods.

2 Calculate the *cg* of a model car or plane.

- **a** Place a postal scale under each wheel. Add the readings of each scale to get the model total weight.
- **b** Now measure the location of the wheel with respect to the nose of the model, i.e., find the arm of each wheel.
- **c** Generate a chart similar to that of Example 3. Divide the total moment by the total weight and get the *cg* location.
- **d** Suspend the model by a string located at the calculated *cg* location and determine if the model is level. If not, remeasure and try again.

Session 2
Weight and Balance

Operational Supplement

Center of Gravity

We often represent the weight of a body by a single force $\overline{w}$, acting downward. Actually, the earth exerts a force of attraction on each particle of a body; the weight of the body results from adding all the forces that act on all of the particles of the body. The weight $\overline{w}$ not only has magnitude and direction, but, it has a line of action which passes through a special point in the body known as the *center of gravity*.

A single force $\overline{F}$ acting vertically upward can be used to support a body of weight $\overline{w}$. The first condition of equilibrium states the vector sum of all the external forces acting on the body must be zero, so the magnitude of the force F equals the weight $\overline{w}$. This condition, however, is not sufficient to ensure equilibrium. The second condition of equilibrium states the vector sum of all the moments which result from these forces, must equal zero. To accomplish this the forces must be equal in magnitude and opposite in direction. When only two forces act on a body, this second condition of equilibrium can be fulfilled only if $\overline{F}$ and $\overline{w}$ act along the same straight line. If the force $\overline{F}$ is applied at any arbitrary point A in the body shown in Figure 1(a), the body will, in general, rotate about point A as an axis and then ultimately come to rest in an orientation which places $\overline{F}$ and $\overline{w}$ along the same line of action as in Figure 1(b). If the body is now supported at some other point B, the body will rotate about point B as an axis and ultimately come to rest in an orientation which again places $\overline{F}$ and $\overline{w}$ along the same line of action as in Figure 1(c). The lines which pass through A and B intersect at a point C which is the center of gravity (cg) of the body. If a single force $\overline{F} = -\overline{w}$ could be applied at the center of gravity C, the body will be in equilibrium no matter how it is oriented as shown in Figure 1(d).

In many cases of practical interest, the position of the center of gravity of a body can be calculated with the aid of a simple theorem that states: *The moment about any axis produced by the weight of the body acting through the center of gravity must equal the sum of the moments about the same axis produced by the weights of the individual particles of the body.*

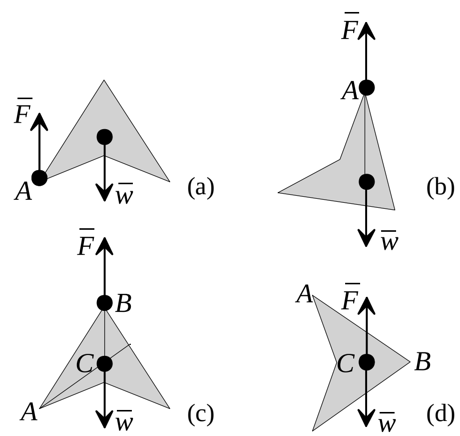


Figure 1 Determining the Center of Gravity

Example 1. Let's assume that we want to know the weight and center of gravity of an empty passenger aircraft sitting on the ground at Kennedy International Airport in New York as depicted in Figure 2. Since the aircraft is at rest, we know that weight of the aircraft is supported by the forces exerted by the pavement beneath each landing gear. We also know from the above theorem that the sum of the clockwise moments produced by each gear about some axis of rotation, say, the tip of the nose of the aircraft, is exactly balanced by the counterclockwise moment produced by the weight of the aircraft acting through the center of gravity. If we pulled the aircraft onto a set of platform scales and measured the gear reaction forces as 100,000 lb for each of the two main gear and 25,000 lb for the nose gear and the distances of the main and nose gear aft of the nose of the aircraft were measured to be 50 ft and 10 ft, respectively, we would get the following results:

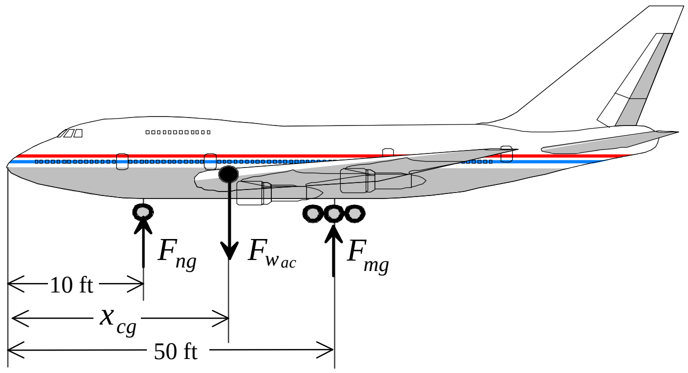


Figure 2 Measuring Weight and Center of Gravity

$$\Sigma\bar{F} = \bar{F}_{w_{ac}} + \bar{F}_{ng} + \bar{F}_{mg} = 0$$

$$w_{ac} = -F_{w_{ac}} = F_{mg} + F_{ng} = 2 \cdot 100{,}000lb + 25{,}000lb = 225{,}000lb$$

$$\Sigma\bar{M} = \bar{M}_{w_{ac}} + \bar{M}_{ng} + \bar{M}_{mg} = 0$$

$$-M_{w_{ac}} = -F_{w_{ac}} \cdot x_{cg} = M_{ng} + M_{mg}$$

$$225{,}000lb \cdot x_{cg} = 2 \cdot 100{,}000lb \cdot 50ft + 25{,}000lb \cdot 10ft = 10.25 \cdot 10^6 ft - lb$$

$$x_{cg} = 10.25 \cdot 10^6 ft - lb/225{,}000lb = 45.56ft$$

We now know that the aircraft, empty of fuel, passengers, and baggage weighs 225,000 lb and has a *cg* 45.56 ft aft of the nose. If we then fuel the aircraft with 40,000 gal of jet fuel weighing 6.25 lb/gal in fuel tanks that have a centroid (center of the volume or mass) location of 40 ft aft of the nose and load the aircraft with 200 passengers weighing an estimated total of 40,000 lb with a centroid of 55 ft aft of the nose and 10,000 lb of baggage in a baggage hold with a centroid of 50 ft aft of the nose, what would be the engine-start gross weight and *cg*? The results are:

$$\bar{F}_{w_{tot}} = \bar{F}_{w_{ac}} + \bar{F}_{w_f} + \bar{F}_{w_p} + \bar{F}_{w_b}$$

$$F_{w_{tot}} = 225{,}000lb + 40{,}000gal \cdot 6.25\tfrac{lb}{gal} + 40{,}000lb + 10{,}000lb = 525{,}000lb$$

$$\bar{M}_{w_{tot}} = \bar{F}_{w_{ac}} \cdot x_{cg} + \bar{F}_{w_f} \cdot x_f + \bar{F}_{w_p} \cdot x_p + \bar{F}_{w_b} \cdot x_b$$

$$M_{w_{tot}} = (225{,}000lb \cdot 45.56lb) + (40{,}000gal \cdot 6.25\tfrac{lb}{gal} \cdot 40ft) + (40{,}000lb \cdot 55ft) + (10{,}000lb \cdot 50ft)$$
$$= 22.95 \cdot 10^6 ft - lb$$

$$x_{cg} = M_{w_{tot}}/F_{w_{tot}} = 22.95 \cdot 10^6 ft - lb/525{,}000lb = 43.71ft$$

So, the fully loaded aircraft has a weight of 525,000 lb and a *cg* 43.71 ft aft of the nose. Notice that the c.g. is at a location forward of the main gear. What would happen if the *cg* was aft of the main gear?

Example 2. Let's take the same fully loaded aircraft in the above example and look at forces acting on the aircraft after takeoff and after it has levelled off at cruise altitude with 4,000 lb of fuel having been burned to get there as shown in Figure 3. The aircraft is in stabilized, level flight, such that the weight of the aircraft is supported by the lift forces being generated by the wing and the tail. If the center of pressure on the wing (where the resultant wing lift force acts) is located at 45.0 ft aft of the nose of the aircraft and the center of pressure of the tail is 105 ft aft of the nose, what are the magnitude and direction of the forces acting on the wing and tail? Again, the sum of the lift forces on the wing and tail are equal and opposite to the weight of the airplane and the sum of the clockwise moments produced by the lift forces about the nose is exactly balanced by the counterclockwise moments produced by the weight of the aircraft acting through the center of gravity. We must first recalculate the weight and center of gravity of the aircraft after 4,000 lb of fuel have been consumed:

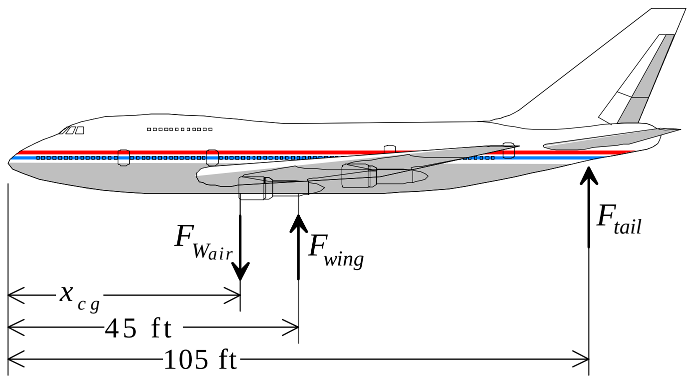


Figure 3 Level Flight Forces and Moments

$$F_{w_{air}} = F_{w_{gnd}} - F_{w_{fu}} = 525,000lb - 4,000gal \cdot 6.25\tfrac{lb}{gal} = 500,000lb$$

$$M_{w_{air}} = F_{w_{air}} \cdot x_{cg} = F_{w_{ac}} \cdot x_{ac} + F_{w_f} \cdot x_f + F_{w_p} \cdot x_p + F_{w_b} \cdot x_b$$

$$M_{w_{air}} = 500,000lb \cdot x_{cg} = 225,000lb \cdot 45.56ft + 36,000gal \cdot 6.25\tfrac{lb}{gal} \cdot 40ft + 40,000lb \cdot 55ft$$
$$+10,000lb \cdot 50ft = 21.95 \cdot 10^6 ft-lb$$

$$x_{cg} = M_{w_{air}}/F_{w_{air}} = 21.95 \cdot 10^6 ft-lb/500,000lb = 43.90ft$$

We must now express the equilibrium conditions of level flight in terms of the force and moment equations:

$$\Sigma\bar{F} = \bar{F}_{w_{air}} + \bar{F}_{wing} + \bar{F}_{tail} = 0$$

$$\Sigma\bar{M} = \bar{F}_{w_{air}} \cdot x_{cg} + \bar{F}_{wing} \cdot x_{wing} + \bar{F}_{tail} \cdot x_{tail} = 0$$

Solving these equations simultaneously for the lift forces on the wing and tail, we get:

$$500,000lb = F_{wing} + F_{tail}$$

$$500,000lb \cdot 43.90ft = F_{wing} \cdot 45ft + F_{tail} \cdot 105ft$$

And by substitution, we can solve for the lift forces on the wing and tail :

$$500,000lb \cdot 43.90ft = F_{wing} \cdot 45ft + (500,000lb - F_{wing}) \cdot 105ft$$

$$F_{wing} = 500,000lb \cdot (105ft - 43.90ft)/(105ft - 45ft) = 509,167lb$$

$$F_{tail} = 500,000lb - 509,167lb = -9,167lb$$

So, the lift force on the wing is 509,167 and the lift force on the tail is - 9,167 (a downward force). This is so, because the center of gravity of the aircraft was forward of the center of pressure of the wing, requiring a counterclockwise moment by the tail to balance the moment equation.